

# King's College Economics Admissions

## Required coursework

Required: A Level/IB Higher Level Mathematics (or its equivalent for those who study in schools which do not offer A Levels or IB)

Note that neither A level economics nor AS Level Further Mathematics is required.

## Admissions assessment

All applicants (independent of the college they apply) are required to take the pre-interview written assessment for Economics at an authorised centre local to them (for a lot of applicants, this will be their school/college).

This is a two-hour exam consisting of two sections. The first section takes 80 minutes and concerns Problem solving and Maths for Economics. The second section, allocated 40 minutes, involves writing an essay on a given topic of economic interest. You must be registered in advance (separately to your UCAS application) to take the assessment. The registration deadline is 15 October 2018. Your assessment centre must register you for the pre-interview assessment; you're not able to register yourself. See the Faculty of Economics Entry Requirements page and the links there for further details. At that page, you can also find specimen and past papers (along with explained answers). Please note that your performance in the pre-interview assessment will not be considered in isolation, but will be taken into account alongside the other elements of your application.

## Interview

If you are invited for an interview at King's, you will have a half-hour interview preceded by a one-hour preparation time. At the beginning of this preparation hour, you will be given a handout which contains three questions of analytical/mathematical flavour, and a recent article from a newspaper. You are allowed to make notes during this hour, and then refer to these notes in the interview. We usually spend the first third of the interview on discussing the reading. Then we move on to discuss your answers/thoughts on the questions from the handout. We may also ask you to solve some new problems during the interview. Note that calculators are not allowed.

See below samples of reading and questions used in our recent admissions interviews.

## Reading

1. "A hated tax but a fair one" from *The Economist*, 23 November 2017.
2. "Will customer ratings for airlines prove as important as those of hotels?" from *The Economist*, 12 July 2016.

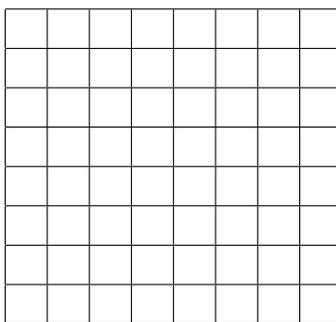
3. “Know your facts: Poverty numbers” by José Cuesta, Mario Negre, Christoph Lakner, from <http://voxeu.org/> 7 November 2016
4. “Abnormally normal – For once, oil prices are responding to supply and demand, not OPEC” from *The Economist*, 14 November 2015.
5. “What happens when a country goes bust?” from *The Economist*, 24 November 2014.
6. “Is Bitcoin about to change the world?” from *The Guardian*, 25 November 2013.

## Questions

1. Does the following function have an extremum? If yes, find it. Is it a max or a min?

$$f(x) = xe^{-x}$$

2. Consider a city on a square grid: 9 parallel streets running east-west. Vertical to these streets, there are 9 other streets running north-south. If we mark the southwest corner on the map as the point  $A = (0, 0)$ , then the northeast corner would be marked  $B = (8, 8)$ .



- (a) What is the length of the shortest path from  $A$  to  $B$ ?
  - (b) Let  $C$  be the street corner  $(5, 3)$ . What is the length of the shortest path from  $A$  to  $B$  that goes through  $C$ ?
  - (c) Two paths are called different from each other if they differ in at least one block. How many different shortest paths are there from  $A$  to  $B$  that go through  $C$ ?
3. (a) Mr and Mrs Smith have two children. One of them is called John. What is the probability that Mr and Mrs Smith have two sons?
  - (b) Mr and Mrs Jones have two children. The youngest child is called Helen. What is the probability that Mr and Mrs Jones have two daughters?

4. Consider the following two-person game. Alice and Bob will take turns. The game will go on for **at most** three stages. At any stage, the player whose turn it is to play can choose to end the game. Therefore the game will end either at the end of stage 1, stage 2 or stage 3.

In stage 1, there are two piles of money on the table: one pile of 1 pound, the other of 4 pounds. Alice, who goes first, has two options:

- pick one of the piles to keep for herself, let Bob have the other pile, and end the game,

or

- move to stage 2.

If Alice has moved to stage 2, both piles are doubled in size. Therefore, one pile has 2 pounds, and the other has 8 pounds. It is now Bob's turn, and Bob has the following two options:

- pick one of the piles to keep for himself, let Alice have the other pile, and end the game

or

- move to stage 3

If Bob has moved to stage 3, both piles are doubled again. One pile has 4 pounds, and the other has 16 pounds. Now Alice gets to choose the pile she likes, Bob gets the other pile, and the game ends.

Assume that Alice *only* cares about how much money she gets to have at the end for herself, and Bob *only* cares about how much money he gets to have at the end for himself.

What do you think will happen in this game?

Stage	Player	Options		
1	Alice	£1	£4	Move to stage 2
2	Bob	£2	£8	Move to stage 3
3	Alice	£4	£16	

*The six-stage version*

Now, imagine a **six-stage version** of the above game. At each stage before the sixth stage, the player whose turn it is to play can decide to either end the game by keeping one of the piles on the table, or to move to the next stage. If the game reaches stage 6, the player whose turn it is to play keeps the pile he/she prefers, and the other pile is kept by the other player.

See below who gets to go at each stage, and the options available at each stage.

Stage	Player	Options		
1	Alice	£1	£4	Move to stage 2
2	Bob	£2	£8	Move to stage 3
3	Alice	£4	£16	Move to stage 4
4	Bob	£8	£32	Move to stage 5
5	Alice	£16	£64	Move to stage 6
6	Bob	£32	£128	

What do you think will happen in this game?

5. At the end of secondary school, all graduating students take the same nation-wide exam. The ministry of education computes the average score in each school and finds that the best performing schools (the schools with highest average scores) tend to be small schools (those with a smaller number of students).

The politicians turn to economists, sociologists and others to help explain why this is so. Upon widespread news coverage, the public demands that big schools split into smaller ones.

However, a statistician warns:

*One doesn't need to look at differences between social backgrounds, or respective teaching qualities of schools of different sizes. Perhaps one can explain this without any social science, and simply by randomness.*

To those who are willing to listen further, he asks

*Why don't you look at the lowest performing schools, too?*

What do you think the statistician means?

6. (a) Three individuals  $A, B, C$  have the following productive opportunities:
- $A$  and  $B$  can form a team to earn a total of £4. That leaves  $C$  with nothing.
  - $A$  and  $C$  can form a team to earn a total of £4. That leaves  $B$  with nothing.
  - $B$  and  $C$  can form a team to earn a total of £4. That leaves  $A$  with nothing.

Before they form a team, they can talk to each other and write binding agreements regarding how the members of a team will share what they produce together. For example,  $A$  and  $B$  can discuss the option of forming a team, let  $A$  keep £1.25 and let  $B$  keep £2.75. To counter that proposal,  $C$  can offer to  $A$  the option: “Form a team with me instead, and let each of us keep £2 each.” All individuals care only about their individual earnings, therefore  $A$  would certainly prefer this new proposal, to the previous one. That is, the previous proposal is **blocked** by a new one.

Is there an “arrangement” which describes who forms a team with whom, and how they share the gains so that no other proposal can block this arrangement? If so, how many different such arrangements are there?

- (b) In addition to the above options, now assume that there is also the option of all three individuals forming a team to earn a total of £6.

Is there an “arrangement” which describes who forms a team with whom, and how they share the gains so that no other proposal can block this arrangement? If so, how many different such arrangements are there?

- (c) Now assume that there are two types of agents: *left-handeds* and *right-handeds*. A left-handed agent and a right-handed agent can get together to earn a total of £2. (And this is the only way a team can earn anything.)
- i. If there is an equal number of both types, which arrangements (team formations and sharing of the gains) can emerge?
  - ii. If there are more right-handed agents than left-handed agents, which arrangements (team formations and sharing of the gains) can emerge?

7. Plot the graph of the function

$$f(x) = \frac{x^3 - 4x}{x^3 + 1}$$

Prepare to explain how you got the features of the plot.

8. Alice and Bob play the following gambling game: There are three fair coins available, a 5p coin, a 10p coin and a 20p coin. Alice owns one of these coins, and Bob owns the other two coins. All three coins are tossed simultaneously. It is agreed that any coin falling tails counts zero for its owner. Any coin falling heads counts its value in pence for its owner. The person that gets the highest score wins all three coins. If all coins come up tails no one wins and the toss is repeated. If at least one coin comes up heads, the game ends.

Does it matter which coin Alice owns?

9. Kate and Ben are on two different trains heading to the same station. Kate’s train will arrive between 3pm and 4pm; its arrival time is random and equally likely for

any moment within that hour. Likewise, Ben's train will arrive between 3pm and 4pm; its arrival time is also random and equally likely for any moment within that hour. Upon arriving, both Kate and Ben will wait at the platform for 15 minutes, and then disappear. What is the probability that Kate and Ben will run into each other on the platform?

10. Everyday, Mr. Jones takes the train from Cambridge to London. Depending on how much time Mr. Jones needs to find his socks in the morning, his arrival time at the station is random – he is equally likely to be at the station at any point between 8am and 9am. Once at the station, Mr. Jones takes the first train that departs to London.

Two companies operate trains from Cambridge to London. Company 1 has a fixed schedule of trains running all day with exactly 20 minutes between its trains departing Cambridge. Likewise, Company 2 has a fixed schedule of trains running all day with exactly 20 minutes between its trains departing Cambridge.

After a year of work Mr Jones suddenly realises that he has used Company 2 three times as many times as he has used Company 1. Provide an example of train schedules which can explain this.

11. A fair coin is tossed but you cannot see whether it is heads or tails. In order to learn the outcome you ask three friends who all see the outcome clearly. Each friend has a habit of lying one third of the time at random. If all of them say that it is heads, what is the probability that it is indeed heads?
12. Five pirates will divide the gold from the latest ship robbery: 100 coins in total. They all have different age. The division happens as follows: the oldest pirate proposes the share of each pirate. Then the five pirates vote on whether to accept the proposal or not. If at least half of the pirates agree, then the gold is divided as proposed. If more than a half rejects the proposal, the proposal is rejected, and the proposer is excluded: he has no longer a vote nor a claim to the gold. The next oldest pirate makes a new proposal to divide the gold among the four remaining pirates. If again more than a half rejects the proposal the next oldest pirate has the right to make a proposal to divide the gold among the remaining three pirates. The procedure continues until the proposal is accepted.

Which pirate will have the most coins?

13. Each person in a room full of people is asked to write an integer between 1 and 99 in a sealed envelope of her own. Then all the envelopes are opened and the average number is calculated. The person who submitted the number closest to the two thirds of this average wins a prize of £100. If you were in the room, what number would you put in your envelope? Explain why. If this was a room full of chess masters, would your answer change? Explain.

14. Three points are randomly drawn on a circle. What is the probability of them being on the same semi-circle? Explain how you reached your conclusion.
15. A *friendly trio* is defined to be a group of three people all of whom know each other. In contrast, define an *awkward trio* to be a group of three people none of whom know any other in the group.

True or false: among any group of six people, there must be a friendly trio or an awkward trio. Justify and explain your answer clearly.

16. A commuter is in the habit of arriving at his local train station each evening at exactly six o'clock. His wife always drives to the station, also to arrive there exactly at six to meet her husband and drive him home.

One day, he takes an earlier train, and arrives at his station at half past five. He decides to walk home along the route always taken by his wife so she can see him on her way to the station. His wife leaves home at her usual time (which means she would arrive at the station at exactly six), but on the way to the station she manages to see her husband walking home. So she stops and lets him in, and they drive back home, arriving twenty minutes earlier than they usually do.

Assuming that the wife always drives with a constant speed, for how long did the husband walk before he was picked up?

17. Ann and Bob are trying to agree on which restaurant to go to. They can check out the menus online before choosing one. Ann is keen to go to restaurant  $A$ . Bob is not sure about  $A$ , for he says: "Look, compared with  $A$ , restaurant  $B$  has five more side dishes on its menu, and steaks are bigger!"

Bob has a very limited memory, and a simple criterion to compare any two restaurants. For example, when comparing two restaurants  $X$  and  $Y$ :

- if the number of side dishes in the menus of  $X$  and  $Y$  differ by at least three, then he prefers the restaurant with more side dishes, and forgets about the other restaurant;
- if, however, the number of side dishes in the two restaurants differ by at most two, then he goes with the restaurant which offers bigger steaks, and forgets about the other restaurant.

Searching online, Ann can find restaurants with any menu she can imagine. Can she persuade Bob to go to restaurant  $A$ ? If yes, explain how. If not, explain why not.

18. A theatre has one hundred seats. All seats are numbered, there are one hundred tickets: one ticket for each seat. All tickets have been sold, and the 100 ticket owners are queuing. Of these 100 people, one of them is drunk, but he is not at the end of the queue (so he is one of the first 99 people in the queue). The ticket holders enter the theatre one by one according to their order in the queue. The

drunk person, upon entering the theatre, arbitrarily chooses one of the unoccupied seats, and sits there. A sober person, on the other hand, goes straight to his assigned seat, and sits there if the seat is not occupied. If his seat is already occupied, then he arbitrarily chooses one of the unoccupied seats, and sits there.

What is the probability that the person at the end of the queue gets to sit at his assigned seat?

19. Mrs Smith wants to buy an exotic plant for her garden, that costs £100. She has a total £10 in cash. There is a bank in the city, in which one can open an account and put the money in. Each morning the bank *doubles* the account balance from the previous business day. Unfortunately, Mrs Smith is too busy to go to the city, so she asks her husband, Mr Smith, to go instead and open an account in her name and deposit money on her behalf. Mr Smith has more free time and can generally go to the city every day to deposit money if needed, but he can only go once a day. However, there is the following complication. On the way to the city, there is a pub, which sells beer for £3 pounds a pint. When Mr Smith is on the way to the city, and has enough money on him, he buys one pint (and no more). Mrs Smith wants to get to £100 in her account as soon as possible, because as soon as the account balance reaches or exceeds £100, she can order the plant online from home, using her debit card.
  - (i) How much money should Mrs Smith give to Mr Smith to deposit to her account on day 1, 2, ... in order to get to the £100 as soon as possible?
  - (ii) In how many days will she then be able to buy the plant?
  - (iii) Do your answers to (i) and (ii) change if the price of a pint of beer is £7?
20. We play a game with two fair dice. Each has six sides. The first die is a usual one, with numbers on its sides from 1 to 6. The second die however is unusual, because it has a different set of numbers on its sides. If we roll the two dice together, the sum of the two numbers will be 1, 2, ..., 12 with equal probability of  $1/12$  each. What are the numbers on the second die?