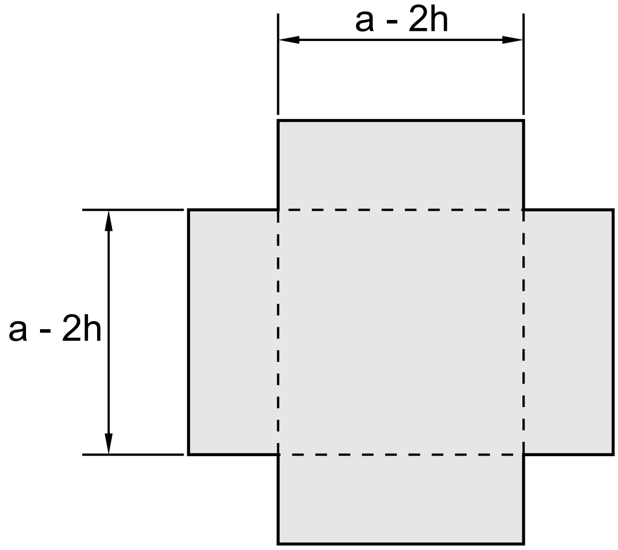
Solution

This kind of optimisation problem is popular at interview as it explores a lot of different engineering concepts. It is a simple application of mathematics to a physical problem to find a real world solution, often with different visualisation and reasoning topics to discuss along the way.

To begin this specific question, you must first construct the expression that gives the volume of the box. For a cuboid this is the base area multiplied by the height. The diagram given in the question is very close to showing this solution already as it has the net of the folded shape. But it is always good to create your own diagram and annotate with useful information either as a reminder to yourself, or to explain your thinking to your interviewers. Something like this is a good place to start, an adapted diagram of the net, showing the side lengths of the box's base



Now the expression for the volume is derived with the three terms shown explicitly for clarity

$$V = h(a - 2h)(a - 2h)$$

To do as the question asks and find the maximum you should then explain how differentiating this expression and setting the result to zero would find the location of the **turning points**. Note, this method doesn't strictly give the location of the maximum right away. Expanding out the expression for the volume

$$V = \ 4h^3 - 4ah^2 + \ a^2h$$

Then differentiating

$$\frac{dV}{dh} = 0 = 12h^2 - 8ah + \ a^2$$

And factorising out, always the neatest and often the fastest method

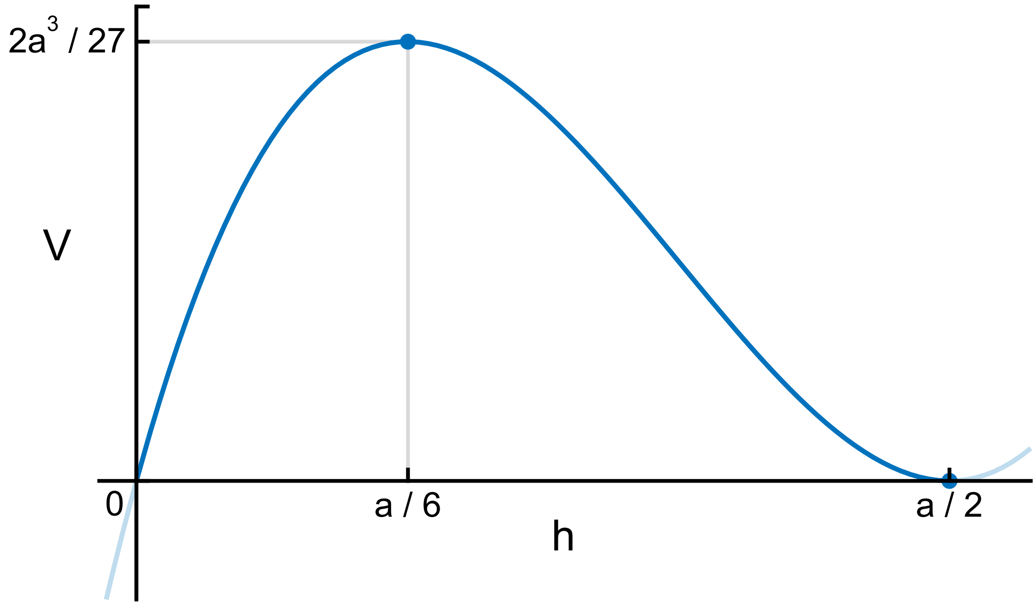
$$(a - 2h)(a - 6h) = 0$$

gives turning points when

$$h = \frac{a}{2} \quad \text{and} \quad h = \frac{a}{6}$$

If you are not sure which to pick at this stage, you could either put the values for the height back into the expression for the volume or proceed to the next stage and consider the graph of the volume.

Looking back at the original volume expression it can be seen from the highest term that it is a positive cubic polynomial. There is no constant and so it must pass through the origin. Given this and the location of the two turning points the approximate shape can be drawn.



Now it is clear that the lower of the two values of h is the maximum. The values can be input into the formula to add the salient point showing the maximum. The other turning point is a minimum and from the formula for V it is clear that the box in this case has zero volume. The original figure with the net shows that if $h = a/2$ then the entire sheet of metal has been cut away and there is nothing left to fold a box from!

The graph is now complete, and the relevant limits are shown. It is not physical or relevant in engineering to consider a graph with negative values of h or values greater than $a/2$, although the shape of the curve can sometimes help you to interpret the turning points.